

# Transilvania University of Braşov, Romania

## Study program: Fundamental Mathematical Structures

Faculty of Mathematics and Computer Science

Study period: 2 years (master)

### 1<sup>st</sup> YEAR

Course title	Code	No. of credits	Number of hours per week			
			course	seminar	laboratory	project
Fix Point Theory	MM11	6	2	1	-	-

**Course description (Syllabus):** Mapping on metric spaces;  $\phi$ -contractions; Generalized  $\phi$ -contractions; Common fixed points and coincidence points; Stability of fixed points; Multivalued mappings.

Course title	Code	No. of credits	Number of hours per week			
			course	seminar	laboratory	project
Riemannian Geometry	MM12	6	2	1	-	-

**Course description (Syllabus):** Riemann manifolds: definitions, examples, gradient, volume form, divergence, Levi-Civita connection, isometries; Geodesics on Riemann manifolds: variational theory of the geodesics, Jacobi fields, exponential map; Sectional curvature of the Riemann manifold: Riemann tensor, Ricci curvature, Einstein spaces; Laplace Operator on a Riemann manifold: harmonic forms, Hodge de Rham Theorem; Riemann submanifolds: classes of submanifolds (umbilical, minimal submanifolds); Compare Theorems: Bonnet, Myers.

Course title	Code	No. of credits	Number of hours per week			
			course	seminar	laboratory	project
Geometric Function Theory	MM13	6	2	1	-	-

**Course description (Syllabus):** Special classes of univalent functions: functions with positive real part, starlike functions, convex functions,  $\alpha$ -convex functions, close-to-convex functions; Integral and differential operators: integral operators, differential operators; Differential subordinations: Schwarz lemma, the method of differential subordinations, applications; Subordination chains: simple univalence criteria, the method of subordination chains, univalence criteria obtained with the method of subordination chains.

Course title	Code	No. of credits	Number of hours per week			
			course	seminar	laboratory	project
Fractals Theory	MM14	6	2	1	-	-

**Course description (Syllabus):** Cantor's triadic set, Sierpinski's triangle, Koch's curve; Takagi-type functions; Topological dimensions; Hausdorff measure and dimension; Code space; Hausdorff-Pompeii metric; Iterative function systems (IFS), the attractor of an IFS; The connection between the set of attractors of IFSs on a complete metric space  $X$  and the set of compacts in  $X$ ; The size of the attractor of an IFS, the parameter dependence of the attractor of an IFS; The attractor connectivity of an IFS; Hutchinson metric, IFSs with probabilities, Hutchinson measure; Fractal interpolation.

Course title	Code	No. of credits	Number of hours per week			
			course	seminar	laboratory	project
Relativity Theory	MM15	6	2	1	-	-

**Course description (Syllabus):** Principles of special relativity. Relativistic interval, Minkowski metric, cone structure. Lorentz transformations and their consequences. Lagrangian of a free particle in special relativity, energy and

momentum. Vectors, covectors, tensors and differential forms on  $R^n$ . Maxwell equations. Differentiable manifolds, pseudo-Riemannian metrics, geodesics, Levi-Civita connection. Principles of general relativity and consequences. Newtonian approximation of the gravitational field. Hilbert action, Einstein field equations. Linearized solutions of vacuum Einstein equations, gravitational waves. Schwarzschild solution, black holes. Elements of cosmology: Friedmann-Lemaitre-Robertson-Walker metric, Big Bang and the expansion of the Universe.

Course title	Code	No. of credits	Number of hours per week			
			course	seminar	laboratory	project
Non-Riemannian Geometry	MM21	7	2	1		-

**Course description (Syllabus):** Lie groups, Lie algebra associated to a Lie group. Local Lie groups. Applications of Lie groups to differential equations. Symplectic and almost symplectic manifolds. Hamiltonians on a symplectic manifold, Hamiltonian vector field, Poisson bracket and applications. Contact manifolds, Reeb vector field. Complex and almost complex manifolds. Hermitian and Kahler manifolds, holomorphic sectional curvature.

Course title	Code	No. of credits	Number of hours per week			
			course	seminar	laboratory	project
Variational Methods	MM22	7	2	2	-	-

**Course description (Syllabus):** Extrema of differentiable functional; Classical variational calculus (Indirect methods); First order variation; Euler's equation; Prime integrals; Second order variation; Legendre's necessary condition; Weierstrass sufficient conditions; Generalizations: Euler-Lagrange system, The Euler-Poisson equation, The Euler-Ostrogradski equation; Dirichlet's variational principle; Isoperimetric problems; Lagrange multiplier; Link to the Sturm-Liouville problem; Functional spaces (Lebesgue spaces: definition and properties (approximation by continuous functions, reflexivity, duality); Sobolev spaces: definition and properties (approximation by smooth functions, reflexivity, duality, Sobolev diving, trace operator)); Non-smooth variational calculus (Direct methods)(Functional energy; The Euler-Ortrogradski equation; The notion of a weak solution; Solution of variational non-smooth problem: Coercivity, Convexity, Weak lower semicontinuity, Existence and uniqueness theorems for minimizers; Non-smooth isoperimetric problems. Lagrange multiplier; Existence theorems for minimizers).

Course title	Code	No. of credits	Number of hours per week			
			course	seminar	laboratory	project
Stochastic Processes and Applications	MM23	7	2	1	-	-

**Course description (Syllabus):** Probability space (sigma-algebra, probability measure, probability space, random variables. Examples and properties). Conditional expectation with respect to a sigma-algebra (filtration, conditional expectation. Examples and properties). Martingale (martingale/submartingale/supermartingale. Examples and properties). Stopping times (definitions, examples, properties). Doob's optional stopping theorem (the case of discrete/continuous martingales and bounded stopping time. Generalizations). Martingale transforms (definitions, examples, properties). Inequalities and convergence theorems (maximal inequalities, upcrossing/downcrossing inequalities theorem, application to the submartingale convergence). Brownian motion. Definition, construction, properties. Stochastic integral (construction, properties). Itô formula (1-dimensional case, extensions to the general case). Probabilistic solution of Dirichlet problem for Laplace's equation. Probabilistic solution of Dirichlet problem for Poisson's equation. Probabilistic solution of heat equation (with Dirichlet / Neumann boundary conditions). Feynman-Kac formula. Black-Merton-Scholes formula.

Course title	Code	No. of credits	Number of hours per week			
			course	seminar	laboratory	project
Theory of Distributions and Applications	MM24	7	2	2	-	-

**Course description (Syllabus):** Fundamental spaces of distributions; The derivative of distributions; the primitive of distributions; Product of distributions; Differential and partial differential equations in distributions; Elements of convex analysis; Banach reflexive spaces; Sobolev spaces; Inferior semi-continuous functions; properties;

Subdifferential of a function; Yoshida approximation and resolvent of multivalued operators; Semi-groups of operators; Equations of evolutions.

Course title	Code	No. of credits	Number of hours per week			
			course	seminar	laboratory	project
Research methodology. Ethics and academic integrity	MM25	2	1	-	-	-

**Course description (Syllabus):** Researchers should promote accuracy, honesty, and truthfulness in their work; Researchers should work in a fair manner taking into account issues of equality, impartiality, and proportionality; Researchers should show respect to the fundamental rights, dignity, and worth of all people.

## 2<sup>st</sup> YEAR

Course title	Code	No. of credits	Number of hours per week			
			course	seminar	laboratory	project
Approximation Theory	MM31	8	2	2	-	-

**Course description (Syllabus):** Density theorems; The theory of best approximation; Jackson's type theorems; Markov's and Bernstein's inequalities; Approximation by positive linear operators; Korovkin type theorems, Approximation by sequences of projectors.

Course title	Code	No. of credits	Number of hours per week			
			course	seminar	laboratory	project
Reliability of Systems	MM32	8	2	1	1	-

**Course description (Syllabus):** Lifetimes; Stochastic orderings; Structure function of coherent systems; Renewal processes; Markov models in reliability; Asymptotic results

Course title	Code	No. of credits	Number of hours per week			
			course	seminar	laboratory	project
Covexity and Inequalities	MM33	8	2	2	-	-

**Course description (Syllabus):** The general theory of convexity, Classical inequalities, Arithmetic inequalities, Inequalities in Hilbert space, Geometric inequalities, Analytic inequalities

Course title	Code	No. of credits	Number of hours per week			
			course	seminar	laboratory	project
Scientific Seminar	MM34	6	-	4	-	-

**Course description (Syllabus):** Presentation of new and significant new results in a certain fields of mathematics: Algorithms and computability, Topological dimension and fractals, Special class of functions in real analysis, Special chapters in the theory of analytic function, Inequalities, Finsler spaces. Activity of documentation; Identification of problems of research and applications. Developing of an own scientific research; Training in writing of scientific papers; Elaboration and presentation of a work at the student scientific session.

Course title	Code	No. of credits	Number of hours per week			
			course	seminar	laboratory	project
Optimal Control	MM41	7	2	1	-	-

**Course description (Syllabus):** Formulation of optimal control problems; Elements of convex analysis; Lower semicontinuous functions, subdifferential; The conjugate function of a convex function; Fenchel's theorem; Linear control problems; Optimal linear control, optimal command; Controllability of linear systems; Pontryagin's principle in the case of optimal linear control; Theorems of the existence of an optimal control in the linear case; Quadratic control

problems; Optimal quadratic control, optimal command; Controllability of quadratic systems; Pontryagin's principle in the quadratic case; The quadratic optimal control synthesis problem.

Course title		Code	No. of credits	Number of hours per week			
				course	seminar	laboratory	project
Special Chapters of Mathematics		MM42	7	2	1	-	-

**Course description (Syllabus):** Polynomials with coefficients in a commutative field; Mathematical induction; Congruences in  $\mathbb{Z}$ , Diophantine linear equations with two variables; Combinatorial and trigonometric identities.

Course title		Code	No. of credits	Number of hours per week			
				course	seminar	laboratory	project
Specialized Practice		MM43	3	-	-	-	2

**Course description (Syllabus):** Making a synthesis of the knowledge obtained in a field of mathematics; Specialized Practice Field is designed to provide students with an advanced perspective of social work practice.

Course title		Code	No. of credits	Number of hours per week			
				course	seminar	laboratory	project
Elaboration of the Master Thesis		MM44	7	-	-	-	4

**Course description (Syllabus):** Making a synthesis of the knowledge obtained in a field of mathematics; Completion of the own contributions to the study; Redaction of the thesis

Course title		Code	No. of credits	Number of hours per week			
				course	seminar	laboratory	project
Optional (1)	Operator Theory	MM45	6	2	1	-	-

**Course description (Syllabus):** Linear continuous operators, Classes of operators, Functional calculus; Approximation by general linear positive operators, Semigroup of operators.

Course title		Code	No. of credits	Number of hours per week			
				course	seminar	laboratory	project
Optional (2)	Lipschitz functions	MM46	6	2	1	-	-

**Course description (Syllabus):** General information on Lipschitz functions; The connection between the class of Lipschitz functions and important classes; Rademacher's Theorem; Extensions for Lipschitz functions; Approximations with Lipschitz functions; Lipschitz function spaces.