An elliptic perspective on the d'Alembert operator from general relativity

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We present a distributional notion of d'Alembertian of a signed time distance function to an achronal set in a metric measure spacetime obeying the timelike measure contraction property. We show precise representation formulas and comparison estimates (both upper and lower bounds). Under a general condition (valid for Finsler spacetimes with curvature bounds), we prove the associated distribution is a signed measure which certifies the integration by parts formula. This expands on the recent elliptic interpretation of the d'Alembertian (e.g. enabling us to give a simplified proof of the splitting theorem) we shortly outline; even in the Lorentzian case, our formulas seem to pioneer its meaning across the timelike cut locus. Two central ingredients our work unifies are the localization paradigm of Cavalletti–Mondino and our recent Lorentzian Sobolev calculus. Partly in collaboration with Robert McCann (University of Toronto), Nicola Gigli, Felix Rott (SISSA Trieste), Shin-ichi Ohta (Osaka University), Clemens Sämann (University of Oxford), Argam Ohanyan, Tobias Beran, Matteo Calisti (University of Vienna).