Generalized Finsler geometry for physics of ferromagnetic crystals and soft biologic tissues

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A theoretical framework with a fundamental basis in generalized Finsler geometry [1-3] is established for modeling large-deformation mechanics of continua [4]. A brief review of prior Finsler geometry-based approaches [1, 5-7] is given. A variational approach to constrained equilibrium states is formulated [2, 3]. Internal state vectors are mathematically linked to deformable director vectors of generalized Finsler geometry and are physically interpreted as vector-valued order parameters of phase-field theory [8, 9]. Evolving microstructures and director vectors affect metric tensors, and thus intrinsic material areas, volumes, strains, free energies, and Euler-Lagrange equations. Spatial gradients of directors, associated with regularized anisotropic microstructure features such as internal boundaries, likewise enter energetic potentials and Euler-Lagrange equations for material response. Applications of the theory to crystalline solids and soft biologic tissues are presented. Analytical and numerical solutions to initial-boundary value problems are discussed. For anisotropic crystals, past work [10, 11] on mechanics of phase transitions, twinning, and fracture is reviewed, followed by recent developments on ferromagnetism [12]. For soft-tissue mechanics, descriptions of stretching and tearing of anisotropic skin tissue [13] and shockwave compression of anisotropic skeletal muscle tissue [14] are reported. Results demonstrate agreement with experimental trends and show how generalized Finsler geometry enriches descriptions of condensed matter physics without introduction of ad-hoc phenomenology or spurious parameters.

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