# Metrizability and Ricci-flatness of Berwald m-Kropina spaces and applications to Finsler gravity

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A Berwald space is a Finsler space that admits a (necessarily unique) torsion-free, metric-compatible linear connection on the tangent bundle, akin to the Levi-Civita connection of a pseudo-Riemannian metric. For any Berwald space, one can therefore ask the natural question of whether its canonical torsion-free metric-compatible connection arises as the Levi-Civita connection of some auxiliary pseudo-Riemannian metric. Simply put: is any Berwald space (pseudo-Riemann) metrizable?

For positive definite Finsler spaces that are smooth on the entire slit tangent bundle the answer, due to Szabo, is well-known and affirmative: the affine connection on any such Berwald space is indeed the Levi-Civita connection of some Riemannian metric. However, in alternative signatures like Lorentzian signature (relevant for applications in gravity), and more generally in situations with weaker smoothness constraints, this question has remained almost completely unexplored.

In this talk, we demonstrate that Szabo's metrization theorem does not extend to this more general setting. More precisely, for the class of m-Kropina metrics, we analyze the question of metrizability in detail and we obtain precise necessary and sufficient conditions for local metrizability. We then classify all Ricci-flat, locally metrizable m-Kropina spaces. Since any Ricci-flat Berwald space is automatically an exact solution to Pfeifer and Wohlfarth's vacuum field equation for Finsler gravity (and coincidentally to several alternative proposals for a vacuum field equation as well), this classification has some implications for Finsler gravity, which we discuss.