Complete Finsler Metrics with $K = 0$ and $S = 0$

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Finsler metrics with $K = 0$ and $S = 0$ are extremal in the sense that they implement the equality case in the Heisenberg-Pauli-Weyl principle and Caffarelli-Kohn-Nirenberg interpolation inequality. Up to now, all the known examples of such metrics are either locally Minkowskian or incomplete. In this talk, I will first describe a class of homogeneous Finsler metrics with $K = 0$. It is easy to see that such metrics are complete and locally Minkowskian, thus they satisfy $S = 0$. Then I will introduce a new method for constructing complete examples that are not locally Minkowskian. There are two key observations in this method. The first one is that rotationally symmetric Finsler metrics with $K = 0$ are in one-to-one correspondence to certain plane vector fields that admit an isochronous center. The second one is a new interpretation of the S-curvature. By combining these two observations, one can find a family of complete Finsler metrics with $K = 0$ and $S = 0$, depending on an arbitrary function of one variable.