Higher-order Hilbert forms

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When in 1966 S.S. Chern reviewed basic progress in Finsler geometry about the mid-dle of last century, his key idea for future research was to replace the traditional un-derlying geometric structure for this theory, the tangent bundle, by the projectivized tangent bundle. Within this understanding, the Finsler function, defining the Finsler variational functional, should be replaced with the Hilbert differential form. In this article we follow similar objectives. Our main goal will be to introduce an analogue of the Hilbert form for higher-order parameter-independent variational func-tionals, depending on curves and 1-dimensional submanifolds of underlying mani-folds. First we recall basic variational theory on higher-order velocity bundles, defined by integrating relevant differential forms, the Lepage forms. Next, integral variational functionals on higher-order velocity bundles satisfying the parameter-ivariance condi-tion are studied. Necessary and sufficient conditions for a Lepage form to generate a parameter-invariant variational functional are established. The Lepage forms satisfy-ing these conditions represent a generalisation of the classical Hilbert form.

References

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