Positive linear operators and convex stochastic orders

Ana-Maria Acu[a], Ioan Raşa[b]

[a] *Lucian Blaga University of Sibiu, Department of Mathematics and Informatics, Romania, e-mail: anamaria.acu@ulbsibiu.ro*

[b] *Technical University of Cluj-Napoca, Faculty of Automation and Computer Science, Department of Mathematics, Romania e-mail: ioan.rasa@math.utcluj.ro*

Abstract

We study inequalities involving convex functions and positive linear operators and interpret them within the framework of convex stochastic order $\leq_{cx}$. Our approach is operatorial in nature and provides a unified view of such inequalities involving several classical approximation processes. By considering a broad class of positive linear operators, we obtain new inequalities that describe dominance between the corresponding induced measures.

We consider the direct problem, showing that the convexity of a function f implies inequalities of the type $L_n f \geq M_n f$, where (L_n) and (M_n) are suitable linear positive operators. We also consider the inverse problem: if

$$\int f\, d\mu \leq \int f\, d\nu$$

for every pair of measures (μ, ν) in a suitable class satisfying $\mu \leq_{cx} \nu$, must f be convex? This inverse problem is particularly challenging. As an illustrative example, we ask whether, for $f \in C[0,\infty)$, the condition $S_t f \leq V_t f$ for all $t > 0$, where S_t and V_t denote the Szász–Mirakjan and classical Baskakov operators respectively, necessarily implies the convexity of f.

References

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