DYNAMICS OF THE TAKAGI FUNCTION

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The Takagi function is a classical example of a continuous nowhere differentiable function. It is defined as

$$T(x) = \sum_{n=0}^{\infty} \frac{\phi(2^n x)}{2^n}, \quad x \in [0,1]$$

where $\phi(x)$ denotes the distance from the point x to the nearest integer.
In this talk, we study the discrete dynamical system generated by the Takagi function, namely the one-dimensional dynamical system given by

$$x_{n+1} = T(x_n), \quad x_0 \in [0,1].$$

Firstly, we describe the behavior of the orbit $(T^n(x_0))_n$ for any seed $x_0 \in [0,1]$. The notation T^n represents the composition of T with itself n times.
The value of the Takagi function at a point can be numerically calculated by a computer. However, this value will always be an approximation of the real value because the computer can only sum a finite number of terms in the series defining the Takagi function. For this reason, it is natural to ask whether numerical solutions can be trusted to represent the real orbits of the discrete dynamical system presented above. This leads us to study the shadowing property for the Takagi function.
This is a joint work with Zoltán Buczolich.